Can the notion of a homogeneous gravitational field be transferred from classical mechanics to the Relativistic Theory of Gravity?

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The generalization of the concept of homogeneous gravitational field from Classical Mechanics was considered in the framework of Einstein's General Relativity by Bogorodskii. In this paper, I look for such a generalization in the framework of the Relativistic Theory of Gravitation. There exist a substantial difference between the solutions in these two theories. Unfortunately, the solution obtained according to the Relativistic Theory of Gravitation can't be accepted because it doesn't fulfill the Causality Principle in this theory. So, it remains *open* in RTG the problem of finding a generalization of the classical concept of homogeneous gravitational field.

1 Introduction

In Newton's Classical Mechanics (CM), the homogeneous gravitational field is the gravitational field which, in every point, has the same gradient of the potential. Such a field is produced by an infinite material plane with the constant surface density of mass. In Section 3, is presented this field in CM.

It's natural to ask if the classical concept of the homogeneous gravitational field can be conserved in the Relativistic Theory of Gravitation (RTG). The source of inspiration in the analysis of this problem is the monograph by Bogorodskii [3], Section 17.

In his monograph, Bogorodskii considers the problem of finding the gravitational field produced by a system of masses uniformly distriduted on a plane, according to Einstein's General Relativity Theory (GRT). In Section 4, I present what Bogorodskii means by homogeneous gravitational field in GTR. But, as we'll see, his solution has an unccountable singularity wich appears without any physical explanation.

The problem of such a homogeneous gravitational field in RTG, has briefly considered by E. Soós and by me in the paper [5], Section 3. In Section 5 of the present paper, I analysed in all details this problem. The solution of the complet system of RTG's Eqs. for the considered problem, differs from Bogorodskii's solution. The obtained solution is regular in its entire domaine of definition but it can't be acceptable like a real gravitational field with physical sense because it doesn't fulfill the Causality Principle (CP) in RTG. So, it remains *open* the problem of finding this field according to RTG.

In section 6, I show that if the solution obtained by me is not rejected using CP, the velocity of some free test particles in the produced field, overpass the velocity of light in vacuum.

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2 RTG's equations and the Causality Principle in RTG

RTG was constructed by Logunov and his co-workers (see [1], [2]) as a field theory of the gravitational field within the framework of Special Relativity Theory (SRT). The Minkowski space-time is a fundamental space that incorporates all physical fields, including gravitation. The line element of this space is:

$$d\sigma^2 = \gamma_{mn}(x)dx^m dx^n, \tag{2.1}$$

where x^m, m =1, 2, 3, 4, is an admissible coordinate system in the underlying Minkowski space-time; $\gamma_{mn}(x)$ are the components of the Minkowskian metric in the assumed coordinate system.

The gravitational field is described by a second order symmetric tensor $\phi^{mn}(x)$, owing to the action of which un effective Riemannian space-time arises.

One of the basic assumption of RTG tells us that the behaviour of matter in the Minkowskian's space -time with metric $\gamma_{mn}(x)$, under the influence of the gravitational field $\phi^{mn}(x)$, is identical to its behaviour in the effective Riemannian space-time with metric $g_{mn}(x)$, determined according to the rules:

$$\tilde{g}^{mn} = \sqrt{-g}g^{mn} = \sqrt{-\gamma}\gamma^{mn} + \sqrt{-\gamma}\phi^{mn}, g = \det(g_{mn}), \gamma = \det(\gamma_{mn}). \tag{2.2}$$

Such interaction of the gravitational field with matter was termed the geometrisation principle of RTG.

The behaviour of the gravitational field is governed by the following differential laws of RTG:

$$R^m_n - \frac{1}{2}\delta^m_n R + \frac{m_g^2}{2}\left(\delta^m_n + g^{mk}\gamma_{kn} - \frac{1}{2}\delta^m_n g^{kl}\gamma_{kl}\right) = 8\pi T^m_n, \tag{2.3}$$

$$D_m\tilde{g}^{mn} = 0, \quad m, n, k, l = 1, 2, 3, 4. \tag{2.4}$$

Here R^m_n is Ricci's tensor corresponding to g_{mn} , $R = R^m_m$ is the scalar curvature, δ^m_n are Kronecker's symbols, m_g is the graviton mass and T^m_n denotes the energy-momentum tensor of the sources of the gravitational field. In (2.4) D_m is the operator of covariant differentiation with respect to the metric γ_{mn}. Eqs. (2.3), (2.4) are covariant under arbitrary coordinate transformations with a nonzero Jacobian. In RTG all field variables depend on the universal spatial-temporal coordinates in the Minkowski space-time. The presence of the mass terms in Eqs. (2.3) makes it possible to unambiguously determine the geometry of space-time and the gravitational field energy-momentum density in the absence of matter. Eqs. (2.4) tell us that a gravitational field can have only the spin states 0 and 2. In the work [2], these Eqs. which determine the polarization states of the field, are consequences of the fact that the source of the gravitational field is the universal conserved density of the energy-momentum tensor of the entire matter including the gravitational field. The graviton mass substantially influences on the Uniserse evolution and alters the nature of gravitational collapse.

In this work, because of the extreme smallness of the graviton mass ($m_g \simeq 10^{-66}$ grams), we will analyse the problem of finding the homogeneous gravitational field in RTG, considering Eqs. (2.3) without mass terms.

Relativistic units are used in all Eqs. .

Eqs. (2.4) can be written in the following form (see [1], Appendix 1):

$$D_m\tilde{g}^{mn} = \tilde{g}^{mn}{}_{,m} + \gamma^n_{mp}\tilde{g}^{mp} = 0, \tag{2.5}$$

where γ^n_{mp} are the components of the metric connection generated by γ_{mn} and the comma is the derivation relative to the involved coordinate. The causality principle (CP) in RTG is presented and analysed by Logunov in [2], Section 6.

According to CP any motion of a pointlike test body must have place within the causality light cone of Minkowski's space-time. According to Logunov's analysis CP will be satisfied if and only if for any isotropic Minkowskian vector u^m , i.e. for any vector u^m satisfying the condition:

$$\gamma_{mn} u^m u^n = 0, \tag{2.6}$$

the metric of the effective Riemannian space-time satisfies the restriction:

$$g_{mn} u^m u^n \leq 0 \tag{2.7}$$

According to CP of RTG only those solutions of the system (2.3), (2.4) can have physical meaning which satisfies the above restriction.

It's important to stress the fact that CP in the above form can be formulated only in RTG, because only in this theory, the space-time is Minkowskian and the gravitational field is described by a second order symmetric tensor field $\phi_{mn}(x)$, x^m being the admissible coordinates in the underlying Minkowskian space-time, x^1, x^2, x^3 being the space-like variables and x^4 being the time-like variable.

3 Homogeneous gravitational field in CM

In CM a gravitational field is named homogeneous if its intensity is a constant magnitude or a picewise constant magnitude. Such a field is generated by a system of masses uniformly distributed on a plane. The connection between the surface density σ of the mass and the acceleration $\mathcal{G}$ due to the produced gravitational field is given by the relation:

$$\mathcal{G} = 2\pi\sigma k > 0, \tag{3.1}$$

k being Newton's gravitational constant.

Choosing the Cartesian axes x and y in the mentioned plane and the axis z perpendicular to this plane, the motion of a free test particle in this gravitational field is governed by Eqs.:

$$\frac{d^2x}{dt^2} = 0, \ \frac{d^2y}{dt^2} = 0 \tag{3.2}$$

$$\frac{d^2z}{d^2t} + \mathcal{G} = 0, \text{ for } z > 0 \quad \text{and} \quad \frac{d^2z}{dt^2} - \mathcal{G} = 0, \text{ for } z < 0, \tag{3.3}$$

where t is the Newtonian time.

It can be also notice that in a non-inertial frame which moves with constant proper acceleration $\mathcal{G}$, the laws of motion of a free test particle have the following form:

$$\frac{d^2x}{dt^2} = 0, \ \frac{d^2y}{dt^2} = 0, \ \frac{d^2z}{dt^2} + \mathcal{G} = 0, \text{ for any } z. \tag{3.4}$$

Here x, y, z represent the coordinates of the particle in the non-inertial frame.

In spite of the formal similarity between the system of Eqs. (3.2), (3.3) and Eqs. (3.4), these two sets of laws are not equivalent. For this, it's sufficient to compare Eqs. (3.3) and $(3.4)_3$. While the laws (3.4) can take, by a transformation of frame, the following form:

$$\frac{d^2X}{dt^2}=0,\ \frac{d^2Y}{dt^2}=0,\ \frac{d^2Z}{dt^2}=0, \text{ for any } Z, \tag{3.5}$$

such an operation it's not possible for the laws (3.2), (3.3). The non-equivalence between these two sets of laws reflects the fact that in the first case we deal with a real gravitational field unlike the second case in which an inertial force is present.

That example shows that even in CM, there is a difference between a real gravitational force produced by a distribution of mass and an inertial force acting in a non-inertial frame.

4 Homogeneous gravitational field in GTR

In his monograph [3], Section 17, Bogorodskii has searched for an answer to the following questions: There exists such a homogeneous gravitational field in GTR? Which is the Riemannian metric that represents in GTR the gravitational field produced by an infinite material plane with constant surface density of mass? His answers at these questions are presented below.

Taking into account the classical results, Bogorodskii looks for the solutions of Einstein's Eqs. in the form:

$$ds^2=-Adx^2-Ady^2-Cdz^2+Ddt^2 \tag{4.1}$$

where A, C, D are positive functions depending only on z.

The energy -momentum tensor of the sources which are uniformely distributed on the plane z=0 is:

$$T^{mn}\equiv 0, \text{for any } z\neq 0. \tag{4.2}$$

For the metric (4.1) and for the expression of the energy-momentum tensor (4.2), Bogorodskii concludes that Einstein's field Eqs. are fulfilled if the unknown functions A, C, D, satisfy the following Eqs.:

$$\begin{gathered}
2\left(\frac{A^{'}}{A}\right)^{'}-\frac{A^{'}C^{'}}{AC}+\frac{A^{'}}{A}\left(\frac{2A^{'}}{A}+\frac{D^{'}}{D}\right)=0,\\
2\left(\frac{D^{'}}{D}\right)^{'}-\frac{C^{'}D^{'}}{CD}+\frac{D^{'}}{D}\left(\frac{2A^{'}}{A}+\frac{D^{'}}{D}\right)=0,\\
\frac{A^{'}}{A}\left(\frac{A^{'}}{A}+\frac{2D^{'}}{D}\right)=0.
\end{gathered} \tag{4.3}$$

Here the prime mark denotes the derivation relative to the coordinate z.

According to the last Eq., there are two possibilities:

$$A^{'}=0 \quad \text{or} \quad \left(\frac{A^{'}}{A}+\frac{2D^{'}}{D}\right)=0. \tag{4.4}$$

In the first case it can be taken $A=1$, because the functions A, C, D are determined up to a constant. In this case, Eq. $(4.3)_1$ is evidentely fulfilled and $(4.3)_2$ becomes:

$$\left(\frac{D^{'}}{D}\right)^{'} - \frac{1}{2}\frac{C^{'}D^{'}}{CD} + \frac{1}{2}\left(\frac{D^{'}}{D}\right)^{2} = 0. \tag{4.5}$$

This yields $C = aD^{-1}D^{'\ 2}$, a being a real constant of integration.

So, for this case the solution of Einstein's Eqs. has the form:

$$A = 1, \quad C = aD^{-1}D^{'2}, \tag{4.6}$$

D being an arbitrary function on z .

For the second case it can be taken $A = D^{-2}$ and Eqs. $(4.3)_1$, $(4.3)_2$ become:

$$\left(\frac{D^{'}}{D}\right)^{'} - \frac{1}{2}\frac{C^{'}D^{'}}{CD} - \frac{3}{2}\left(\frac{D^{'}}{D}\right)^{2} = 0. \tag{4.7}$$

This yields $C = bD^{-5}D'^{\ 2}$, b being a constant of integration.

Thus for the second case, the solution of Einstein's Eqs. has the form:

$$A = D^{-2},\, C = bD^{-5}D'^{2}, \tag{4.8}$$

D being an arbitrary function on z .

With the view of finding $D(z)$, Bogorodskii observes that the motion of a free test particle in the produced gravitational field is determined by Eqs. of geodesics:

$$\frac{d^2x}{dt^2} + \left(\frac{A'}{A} - \frac{D'}{D}\right)\frac{dx}{dt}\frac{dz}{dt} = 0,\, \frac{d^2y}{dt^2} + \left(\frac{A'}{A} - \frac{D'}{D}\right)\frac{dy}{dt}\frac{dz}{dt} = 0, \tag{4.9}$$

$$\frac{d^2z}{dt^2} - \frac{A'}{2C}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right] + \left(\frac{C'}{2C} - \frac{D'}{D}\right)\left(\frac{dz}{dt}\right)^2 + \frac{D'}{2C} = 0. \tag{4.10}$$

From the above system it can be seen that the vertical motion, with the velocity equals to zero at the initial moment, is described by the system of Eqs.:

$$\frac{d^2x}{dt^2} = 0,\, \frac{d^2y}{dt^2} = 0, \tag{4.11}$$

$$\frac{d^2z}{dt^2} + \left(\frac{C'}{2C} - \frac{D'}{D}\right)\left(\frac{dz}{dt}\right)^2 + \frac{D'}{2C} = 0. \tag{4.12}$$

In the case of slow motion, the term which contains the velocity must be omitted, so, Eq.(4.12) becomes :

$$\frac{d^2z}{dt^2} + \frac{D'}{2C} = 0. \tag{4.13}$$

Comparing the system of Eqs. (4.11), (4.13) with the classical one, Bogorodskii requires:

$$\frac{D'}{2C} = \mathcal{G}. \tag{4.14}$$

I'll return at this condition.

Finally, taking the constants a, b equal to $\frac{1}{4\mathcal{G}^2}$ and using the relation (4.14), from (4.6), (4.8) Bogorodskii obtaines the solutions:

$$A = 1,\, C = e^{2\mathcal{G}z},\, D = e^{2\mathcal{G}z}, \tag{4.15}$$

and

$$A = (1 - 8\mathcal{G}z)^{1/2},\, C = (1 - 8\mathcal{G}z)^{-5/4},\, D = (1 - 8\mathcal{G}z)^{-1/4}. \tag{4.16}$$

For the first solution (4.15), the Riemann-Christoffel curvature tensor is identically zero. Thus, the autor concludes that (4.15) does not represent a real gravitational field. It's easy to see that by the transformations:

$$X = x,\, Y = y,\, Z = \frac{1}{\mathcal{G}}[e^{\mathcal{G}z} ch(\mathcal{G}t) - 1],\, T = \frac{1}{\mathcal{G}} e^{\mathcal{G}z} sh(\mathcal{G}t), \tag{4.17}$$

the fundamental invariant (4.1) becomes the Minkowskian one:

$$d\sigma^2 = -dX^2 - dY^2 - dZ^2 + dT^2. \tag{4.18}$$

The properties and the singularities of the non-inertial frame characterized by the relations (4.17) are studied in detail in the monograph of Jukov [4], Section15. I don't present here these properties.

Consequently, the first solution (4.15) corresponds to a non-inertial frame whose origine moves with the constant proper acceleration $\mathcal{G}$ along the pozitive axis Z of an inertial frame.

The Riemann-Christoffel curvature tensor corresponding to the second solution (4.16) is not zero. According to Bogorodskii, this solution represents the real homogeneous gravitational field in GRT, produced by the considered distribution of mass.

First of all, it can be observed that the solution (4.16) has a strange singularity in $z = \frac{1}{8\mathcal{G}}$, which is difficult to be explained.

Now, it's the moment to return to the condition (4.14) required by Bogorodskii. It has been pointed out in Section 3, that in CM the motion of a free test particle in the real gravitational field is governed by the system of Eqs. (3.2), (3.3) not by the system (3.4). Thus, Bogorodskii's relation (4.14) must be replaced with:

$$\frac{D'}{2C} = \begin{cases} \mathcal{G} & , z > 0 \\ -\mathcal{G} & , z < 0 \end{cases} \tag{4.19}$$

Hence, Bogorodskii's solution (3.12) must be replaced with:

$$A = (1 \mp 8\mathcal{G}z)^{1/2},\, C = (1 \mp 8\mathcal{G}z)^{-5/4},\, D = (1 \mp 8\mathcal{G}z)^{-1/4}, \tag{4.20}$$

the sign + corresponds to $z < 0$, the sign - corresponds to $z > 0$.

The solution (4.20) can be accepted only for $-\frac{1}{8\mathcal{G}} < z < \frac{1}{8\mathcal{G}}$ and the singularities in $z = \pm\frac{1}{8\mathcal{G}}$ still remain without any physical explanation.

5 Homogeneous gravitational field in RTG

The problem of finding the homogeneous gravitational field according to RTG was shortly considered by E. Soós and by me in the paper [5], Section 3. I present here all details concerning this problem.

To study any problem in RTG's framework, one must solve Eqs. (2.3), (2.4) in terms of the coordinates of the underlying Minkowski space-time. Only those solutions that satisfiy CP could represent physical acceptable gravitational fields.

I keep the same point of departure as Bogorodskii. So, I look for the solutions of this problem in RTG in the form (4.1). Two approaches can be employed in obtaining these solutions. The both will be presented.

At first, I use the already found solutions, (4.15), (4.20), which fulfill Eqs. (2.3). I verify if these solutions satisfy Eqs. (2.4).

For the solution (4.15), the components of the underlying Minkowskian metric and the components of the Riemannian effective metric coincide because this solution appears at the transition from an inertial reference system to an accelerated reference system in Minkowski's space-time.Thus, Eqs. (2.4) are obviously satisfied, D_m being the operator of covariant differentiation with respect to the Minkowskian metric.

For the solution (4.20) , first of all, it must be establish the reference system of the underlying Minkowski space-time. This reference system can be obtained assuming the vanishment of the gravitational field. In this way, for σ=0 and implicitly $\mathcal{G}$=0, the metric (4.1) in the above reference system becomes:

$$d\sigma^2 = -dx^2 - dy^2 - dz^2 + dt^2. \tag{5.1}$$

Consequently, for the chosen system of coordinates, the components of the metric connection γ^n_{mp} are zero and Eqs. (2.5) have the simple form:

$$\tilde{g}^{mn}{}_{,m} = 0. \tag{5.2}$$

Taking into account (2.2), (4.1), (4.20), Eqs. (5.2) are not fulfilled. So, (4.20) is not an admissible solution in RTG. For finding an admissible solution in RTG, I use the same procedure as Logunov and Mestvirishvili in [1], Chapter 13. Thus, I am looking for a system of coordinates $\{\eta^i\}$= $\{X,\ Y,\ Z,\ T\}$ in which Eqs. (2.3) are fulfilled, Eqs. (2.4) establishing a one to one relationship between the sets of coordinates $\{\eta^i\}$ and $\{\xi^i\}$=$\{x,\ y,\ z,\ t\}$ in the Minkowski space-time. This change is made in such a way that, when the gravitational field is swiched off, we arrive in the Minkowski space-time with Galilean metric:

$$d\sigma^2 = -dX^2 - dY^2 - dZ^2 + dT^2. \tag{5.3}$$

Thus, γ^n_{mp} in this system of coordinates are identically null. Because the components of the metric (4.1) depend only on z, I shift from the variables $\{\xi^i\}$ to the variables $\{\eta^i\}$ assuming that:

$$X = x, Y = y, Z = Z(z), T = t. \tag{5.4}$$

We write Eqs. (2.4) in the chosen system of coordinates, in a somewhat different form (see the relations (13.17), (13.22), from [1]):

$$\frac{\partial}{\partial \xi^m}\left(\sqrt{-g(\xi)}g^{mn}(\xi)\frac{\partial \eta^p}{\partial \xi^n}\right) = 0. \tag{5.5}$$

For the transformation (5.4), taking into account (4.1), (4.20), the system (5.5) becomes:

$$\frac{d}{dz}\left((1 \mp 8\mathcal{G}z)\,\frac{dZ}{dz}\right) = 0. \tag{5.6}$$

Integrating this Eq. and choosing the constant of integration, in such a way that for $\mathcal{G}$ converges to zero, Z converges to z,we get:

$$Z = \mp\frac{1}{8\mathcal{G}}\ln\left(1 \mp 8\mathcal{G}z\right). \tag{5.7}$$

Now, it's sufficient to employ the tensor transformation law and we find the components of the metric (4.1), in this second case, in the the system of coordinates (5.4), (5.7):

$$A = e^{-4\mathcal{G}Z},\ C = e^{-6\mathcal{G}Z},\ D = e^{2\mathcal{G}Z},\ \text{for any } Z > 0$$

$$A = e^{4\mathcal{G}Z},\ C = e^{6\mathcal{G}Z},\ D = e^{-2\mathcal{G}Z},\ \text{for any } Z < 0 \tag{5.8}$$

The solution (5.8) satisfies the complet system of Eqs. (2.3), (2.4). This solution is regular for any $Z \neq 0$. Anyway, it is not derivable for Z=0 and crossing this plane the derivatives of the functions A, C, D have finite jumps. Of course, the appearence of this singularity, concentrated in the plane Z=0, reflects the fact that this real gravitational field has like source a system of masses distributed on the respective plane. We observe also that the condition (4.19) can be only approximately fulfilled, because, for example, from $(5.8)_1$:

$$\frac{D'}{2C} = \mathcal{G}e^{8\mathcal{G}Z},\ \text{for any } Z > 0. \tag{5.9}$$

As the same time, we notice that this approximation is justified somehow. If we use the usual units, we get:

$$\frac{D'}{2C} = \frac{\mathcal{G}}{c^2}e^{\frac{8\mathcal{G}Z}{c^2}},\ \text{for any } Z > 0, \tag{5.10}$$

c being the speed of light in vacuum, relative to an inertial frame. Consequently, for any $Z > 0$ with:

$$Z \ll \frac{c^2}{\mathcal{G}}, \tag{5.11}$$

the ratio $\frac{D'}{2C}$ can be considered approximately constant.

The analysis in RTG can't be stoped here, since the solutions must be also submitted to CP.

For the first solution (4.15), CP is fulfilled evidently, because in this case, as it was already mentioned, the Minkowskian metric and the Riemannian metric are the same. Thus, the solution (4.15) is also an admissible solution in RTG. Its physical significance was already clarified.

For the second solution (5.8), taking into account the form (5.3) of the underlying Minkowski space-time, u=(1, 0, 0,1) is an isotropic Minkowskian vector. Consequently, the condition(2.7) is fulfilled if:

$$e^{2\mathcal{G}Z} \leq e^{-4\mathcal{G}Z},\ \text{for } Z > 0 \quad \text{and} \quad e^{-2\mathcal{G}Z} \leq e^{4\mathcal{G}Z},\ \text{for } Z < 0. \tag{5.12}$$

These conditions are not fulfilled for any $Z > 0$ or $Z < 0$, if $\mathcal{G}$ is not zero.

We conclude that in accordance with RTG, can't exist a generalization of the homogenouse gravitational field in Bogorodskii's sense.

It can also be employed the following approache for finding the solution of the considered problem in RTG. From Eqs. (2.3) with $T^m_n \equiv 0$, for $z \neq 0$, we get the solutions (4.6), (4.8). I stress that in (4.6), (4.8), $D(z)$ is an arbitrary function. This fact shows clearly that Einstein's field Eqs., are not enough for finding in an unique manner the gravitational field produced by the considered distridution of mass. I determine the unknown function $D(z)$, using Eqs. (2.4).

Let us consider x, y, z, t the Galilean coordinates of an inertial frame. So, Eqs. (2.4) take the simple forme (5.2). Taking into account (2.2), (4.1), (4.6), (4.8), from (5.2) we get:

$$D(z) = pe^{qz}, \tag{5.13}$$

where p and q are real constants.

Thus, introducing (5.13) into (4.6), we obtain the first solution according to RTG:

$$A = 1,\ C = ape^{qz},\ D = pe^{qz} \tag{5.14}$$

and from (4.8) the second solution:

$$A = p^{-2}e^{-2qz},\ C = bp^{-3}e^{-3qz},\ D = pe^{qz}. \tag{5.15}$$

For the first solution (5.14), the Riemann-Christoffel curvature tensor is identically zero and for the second solution (5.15) it's different from zero.

The constants a, b, p, q must be determined from the Correspondence Principle: after switching off the gravitational field, the curvature of space disappears and we find ourselves in the Minkowski space-time in the chosen reference system. So, Eqs. of motion become classical Eqs. of motion in the chosen reference system. From the geometrization principle of RTG, Eqs. of motion in the considered gravitational field are given by Eqs. (4.9), (4.10). In the case of vertical and slow motion, this system of Eqs. become the system (4.11), (4.13). Thus, from the Correspondence Principle, we must have the relation (4.14) for the solution (5.14) and the relation (4.19) for the solution (5.15).

For the same principle, the metric must tend to the Galilean metric for $\mathcal{G}$ converges to zero. Thus, in the first case, we get the solution (4.15) and in the second case the solution (5.8). For this last solution the relation (4.19) is only approximately fulfilled, as we have already discused.

So, by the two approaches, we have obtained the same result: one solution, (4.15), that represents an inertial force and the other, (5.8), which can't be accepted as the real homogeneous gravitational field produced by the considered distribution of mass, because it doesn't fulfill CP (see 5.12).

6 The importance of CP in RTG

The example which will be presented in this Section shows clearly the importance of CP in deciding the rejection of the solution (5.8). I'll show that there exist free test particles, in the obtained space-time (5.8), which move quickly than the light in vacuum.

Let us consider a free test particle, situated at the initial moment $t = 0$, at the distance $h > 0$ from the plane z=0, where the masses are concentrated. For the sake of simplicity, we consider the problem in the plane xOz. At the initial moment t=0, when the particle has the assumed position:

$$x(0) = 0 \tag{6.1}$$

$$z(0) = h > 0 \tag{6.2}$$

it is thrown up.

From the geometrization principle, if we want to study the behavior of this particle under the influences of the considered homogeneous gravitational field, we can study its behaviour in the following effective Riemannian space-time (see $(5.8)_1$):

$$ds^2 = -e^{-4\mathcal{G}z}dx^2 - e^{-6\mathcal{G}z}dz^2 + e^{2\mathcal{G}z}dt^2 \tag{6.3}$$

Taking into account Eqs. $(4.9)_1$, (4.10) of geodesics and the expression (6.3) of the Riemannian metric, the trajectories of the particle are described by:

$$\frac{d^2x}{dt^2} - 6\mathcal{G}\frac{dx}{dt}\frac{dz}{dt} = 0, \tag{6.4}$$

$$\frac{d^2z}{dt^2} + 2\mathcal{G}e^{2\mathcal{G}z}\left(\frac{dx}{dt}\right)^2 - 5\mathcal{G}\left(\frac{dz}{dt}\right)^2 + \mathcal{G}e^{8\mathcal{G}z} = 0. \tag{6.5}$$

We also consider that at the initial moment t=0:

$$\dot{x}(0) = a > 0, \tag{6.6}$$

$$\dot{z}(0) = b > 0, \tag{6.7}$$

a, b beaing real constants.

Solving Eq. (6.4) and taking into account (6.2), (6.6), we get:

$$\dot{x}(t) = ae^{6\mathcal{G}(z-h)}. \tag{6.8}$$

Also solving Eq. (6.5), on the basis of (6.2), (6.6), (6.7), we get:

$$\dot{z}(t) = e^{4\mathcal{G}z}\sqrt{1 + Le^{2\mathcal{G}z} - a^2e^{-12\mathcal{G}h}e^{6\mathcal{G}z}.} \tag{6.9}$$

Here L is a real constant equals to:

$$L = b^2e^{-10\mathcal{G}h} + a^2e^{-8\mathcal{G}h} - e^{-2\mathcal{G}h}. \tag{6.10}$$

Now, we single out in (6.3) the time-like part and the space-time part:

$$ds^2 = d\sigma^2 - dl^2, \tag{6.11}$$

where:

$$d\sigma^2 = e^{2\mathcal{G}z}dt^2, \tag{6.12}$$

$$dl^2 = e^{-4\mathcal{G}z}dx^2 + e^{-6\mathcal{G}z}dz^2. \tag{6.13}$$

So, if a pointlike event has coordinates $(x,\,0,\,z,\,t)$ and another pointlike event has coordinates $(x+dx,\,0,\,z+dz,\,t+dt)$, then an observer in $(x,\,0,\,z,\,t)$ with four-velocity $(\frac{dx}{ds},\,0,\,\frac{dz}{ds},\,\frac{dt}{ds})$, measures between the two events a proper spatial distance dl and an interval of proper time $d\sigma$.

The velocity $v(v^1 = \frac{dx}{dt},\,0,\,v^3 = \frac{dz}{dt})$ of the particle, in the considered effective Riemannian space-time, has the following absolute value:

$$v^2 = \frac{dl^2}{dt^2} = e^{-4\mathcal{G}z}\dot{x}^2 + e^{-6\mathcal{G}z}\dot{z}^2 \tag{6.14}$$

It is natural to demand that the initial velocity of our particle be smaller than the velocity of light in vacuum. So, from (6.2), (6.6), (6.7), (6.14), we get:

$$e^{-4\mathcal{G}h}a^2 + e^{-6\mathcal{G}h}b^2 < 1. \tag{6.15}$$

Taking into account (6.10), (6.15), we obtain the following restriction for the constant L :

$$L < e^{-4\mathcal{G}h} - e^{-2\mathcal{G}h} \tag{6.16}$$

We have considered $h > 0$, so, from (6.16):

$$L < 0. \tag{6.17}$$

Now, I show that for the case chosen by me, there are some coordinates z, such that the velocity of the particle at these points overpass the velocity of light in vacuum.So, I find some coordinate z such that:

$$v^2(z) > 1. \tag{6.18}$$

Introducing (6.8), (6.9) into (6.14), the inequality (6.18) is equivalent to:

$$e^{2\mathcal{G}z} + Le^{4\mathcal{G}z} > 1. \tag{6.19}$$

Taking into account (6.17), by some calculus, we obtain that for:

$$-\frac{1}{4} < L < 0, \tag{6.20}$$

so, for some conditions imposed on the initial values of the velocity, for any z satisfying:

$$\frac{1}{2\mathcal{G}} \ln\left(\frac{-1+\sqrt{1+4L}}{2L}\right) < z < \frac{1}{2\mathcal{G}} \ln\left(\frac{-1-\sqrt{1+4L}}{2L}\right), \tag{6.21}$$

the inequality (6.19) is fulfilled.

For example, the restrictions (6.20) are fulfilled for the following a and b:

$$\begin{aligned} a &= \rho e^{2\mathcal{G}h}\cos\theta, \\ b &= \rho e^{3\mathcal{G}h}\sin\theta, \text{with } \theta \in (0, \pi/2), 0 < \rho < 1, \rho^2 > 1 - \frac{(e^{2\mathcal{G}h} - 2)^2}{4}. \end{aligned}$$

Hence, if we consider in the effective Riemannian space-time $(5.8)_1$ a free test particle, which has at the initial time the position (6.1), (6.2) and the velocity (6.6), (6.7), such that, taking into account (6.10), the real constants a, b satisfy the restrictions (6.20), this particle moves quikly than the light in vacuum.

7 Conclusions

As we have seen, in CM, GRT, RTG, if a frame is moving at a constant proper acceleration, relative to an inertial frame, then the inertial field due to the force of inertia, is a constant field. The expression of the Minkowski line element (4.15) is the same in GRT and RTG. In CM, it was considered that this constant field is indistingushable from a homogeneous gravitational

field produced by an infinite material plane. But from (3.2), (3.3) and (3.4) we have seen that there exists a difference, even in CM, between these two fields. This is due to the fact that the gravitational force is a force of attraction. In GRT and RTG the difference between the constant field produced by an inertial force and the homogeneous gravitational field due to the presence of mass, is substantial. Bogorodskii's solution in GRT, for the homogeneous gravitational field, is given by (4.16). As we have already discused, his solution has an unccountable singularity. The solution obtained in RTG has the form (5.8). Unfortunately, this solution can't be kept because it doesn't satisfy CP in RTG. The decision of the rejection of this solution, using CP, it's right. Indeed, in the obtained space-time (5.8), the velocity of a free test particle can overpass the velocity of light in vacuum. Ending to this analysis, I can conclude that the problem of finding in RTG the gravitational field produced by a uniform distribution of mass concentrated on an infinite plane is a very interesting problem, but which remains *open*.

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